N70 33297

NASA TECHNICAL TRANSLATION NASA TT F - 13,008

CALCULATION OF TURBULENT FLOW CHARACTERISTICS IN A CHANNEL WITH AN INTERNAL ROTATING CYLINDER

B. P. Ustimenko

Translation of "K Raschetu Turbulentnykh Kharakteristik Techeniya V Kanale S Vnutrennim Vrashchayushchimsya Tsilindrom," Zhurnal Prikladnoy Mekhaniki i Tekhnicheskoy Fiziki, No. 3, 1969, pp. 121-127

CASE FILE COPY

NATIONAL AERONAUTICS AND SPACE ADMINISTRATION
WASHINGTON, D. C. 20546 JUNE 1970

NASA TT F - 13,008

CALCULATION OF TURBULENT FLOW CHARACTERISTICS IN A CHANNEL WITH AN INTERNAL ROTATING CYLINDER

B. P. Ustimenko

ABSTRACT. A theoretical analysis of the turbulent characteristics of circular flow in a channel with an internal rotating cylinder is presented, based on the use of balance equations for pulsating energy for various directions of motion (whole and individual velocity components), along with the ordinary equations of average motion. Examination of the supplementary equations leads to information particularly on the distribution not only of average characteristics in the flow, but of pulsating ones as well. The results of the calculation are compared with experimental distributions of average velocity, pulse intensity, correlation, etc.

This report describes an attempt to analyze theoretically the turbulent /121* characteristics of circular flow in a channel with an internal rotating cylinder, based on the use of equilibrium equations for pulsating energy in various directions of motion. Calculated results are brought into comparison with the experimental distribution of average velocity, pulse intensity, correlation, etc.

A number of papers have been written on the theoretical analysis of turbulent current in channels between two coaxial rotating cylinders. The majority are based on the supposition of small channel curvature [1, 2], and the others [3-5] use various methods of approximation. These works permit relatively satisfactory calculation of the distribution of average velocity in the stream and of friction.

More complete information can be obtained if we use equilibrium equations of pulsating energy (of the whole and of individual velocity components) along with the ordinary equations of average motion which are used. Examination of these supplementary equations makes it possible to obtain information particularly about the distribution of not only the average characteristics in the stream, but also of the pulsating ones.

*Numbers in the margin indicate pagination in the foreign text.

1. Basic Equations. We will examine a plane circular turbulent flow of incompressible liquid, statistically uniform on cylindrical surfaces having a constant radius. For such a flow, the following relationships are fulfilled:

$$v_x = v_x', \quad v_r = v_r', \quad v_\varphi = \langle v_\varphi \rangle + v_\varphi', \quad p = \langle p \rangle + p'$$
$$\langle v_x \rangle = \langle v_r \rangle = 0, \quad \langle v \rangle_\varphi = \langle v_\varphi (r) \rangle$$
$$\langle p \rangle = \langle p (r) \rangle$$

while the derivatives for coordinates x and ϕ from the averaged values equal zero. Here $\langle v_x \rangle$, $\langle v_r \rangle$, $\langle v_\phi \rangle$, $\langle p \rangle$, v_x', v_r', v_ϕ', p are the averaged and pulsating values of the axial, radial and tangential components of the vector of velocity and static pressure, is the sign of average with time (per the Reynolds concept).

Accounting for these expressions the system of differential equations for the correlative tensor component $\langle v_i' v_j' \rangle$ will have the form

$$\left\langle \frac{1}{\rho} p' \frac{\partial v_x'}{\partial x} \right\rangle - \nu \left[\left\langle \left(\frac{\partial v_x'}{\partial x} \right)^2 \right\rangle + \left\langle \left(\frac{\partial v_x'}{\partial r} \right)^2 \right\rangle + \left\langle \left(\frac{\partial v_x'}{r \partial \varphi} \right)^2 \right\rangle \right] = 0 \tag{1.1}$$

$$2 \langle v_r' v_\varphi' \rangle \frac{\langle v_\varphi \rangle}{r} + \left\langle \frac{1}{\rho} p' \frac{\partial v_r'}{\partial r} \right\rangle - \nu \left[\left\langle \left(\frac{\partial v_r'}{\partial x} \right)^2 \right\rangle + \left\langle \left(\frac{\partial v_r'}{\partial r} \right)^2 \right\rangle + \left\langle \left(\frac{\partial v_r'}{r \partial \varphi} \right)^2 \right\rangle \right] = 0 \tag{1.2}$$

$$- \langle v_r' v_\varphi' \rangle \frac{1}{r} \frac{d}{dr} (\langle v_\varphi \rangle r) + \left\langle \frac{1}{\rho} p' \frac{\partial v_\varphi'}{r \partial \varphi} \right\rangle - \nu \left[\left\langle \left(\frac{\partial v_\varphi'}{\partial x} \right)^2 \right\rangle + \left\langle \left(\frac{\partial v_\varphi'}{\partial r} \right)^2 \right\rangle + \left\langle \left(\frac{\partial v_\varphi'}{r \partial \varphi} \right)^2 \right] = 0 \tag{1.3}$$

$$\langle v_x' v_\varphi' \rangle \frac{2 \langle v_\varphi \rangle}{r} + \left\langle \frac{1}{\rho} p' \left(\frac{\partial v_r'}{\partial x} + \frac{\partial v_x'}{\partial r} \right) \right\rangle - 2\nu \left[\left\langle \frac{\partial v_x'}{\partial x} \frac{\partial v_r'}{\partial x} \right\rangle + \left\langle \frac{\partial v_x'}{\partial r} \frac{\partial v_r'}{\partial r} \right\rangle + \left\langle \frac{\partial v_x'}{\partial r \varphi} \frac{\partial v_r'}{r \partial \varphi} \right\rangle \right] = 0 \tag{1.4}$$

$$- \langle v_x' v_r' \rangle \frac{1}{r} \frac{d}{dr} (\langle v_\varphi \rangle r) + \left\langle \frac{1}{\rho} p' \left(\frac{\partial v_\varphi'}{\partial x} + \frac{\partial v_x'}{r \partial \varphi} \right) \right\rangle - 2\nu \left[\left\langle \frac{\partial v_x'}{\partial x} \frac{\partial v_\varphi'}{\partial x} \right\rangle + \left\langle \frac{\partial v_x'}{\partial r} \frac{\partial v_\varphi'}{\partial r} \right\rangle + \left\langle \frac{\partial v_x'}{r \partial \varphi} \frac{\partial v_\varphi'}{r \partial \varphi} \right\rangle \right] = 0 \tag{1.5}$$

$$- \langle v_r'^2 \rangle \frac{1}{r} \frac{d}{dr} (\langle v_\varphi \rangle r) + \langle v_\varphi'^2 \rangle \frac{2 \langle v_\varphi \rangle}{r} + \left\langle \frac{1}{\rho} p' \left(\frac{\partial v_\varphi'}{\partial r} + \frac{\partial v_r'}{r \partial \varphi} \right) \right\rangle - 2\nu \left[\left\langle \frac{\partial v_r'}{\partial x} \frac{\partial v_\varphi'}{\partial x} \right\rangle + \left\langle \frac{\partial v_r'}{\partial r} \frac{\partial v_\varphi'}{\partial r} \right\rangle + \left\langle \frac{\partial v_r'}{r \partial \varphi} \frac{\partial v_\varphi'}{r \partial \varphi} \right\rangle \right] = 0 \tag{1.6}$$

In the system of equations (1.1) through (1.6), terms related to convection transfer of turbulent energy by average motion are dropped, and also those related to viscous and turbulent diffusion [6-8].

Following [6], using the approximate semi-empirical relationship for dissipation of pulsating motion, /122

$$D = 2\nu \sum_{k=1}^{3} \left\langle \frac{\partial v_i'}{\partial x_k} \frac{\partial v_j'}{\partial x_k} \right\rangle = \nu c_1 \frac{\langle v_i' v_j' \rangle}{l^2} + \delta_{ij} \frac{2c}{3} \frac{E^{3/2}}{l} \tag{1.7}$$

and for energy exchange between different pulsation components

$$\frac{1}{\rho} \left\langle p' \left(\frac{\partial v_i'}{\partial x_i} + \frac{\partial v_j'}{\partial x_i} \right) \right\rangle = -k \frac{\sqrt{E}}{l} \left(\langle v_i' v_j' \rangle - \delta_{ij} \frac{2}{3} E \right), \quad E = \sum_{i=1}^{3} \frac{\langle v_i'^2 \rangle}{2} \tag{1.8}$$

Here E is the kinetic energy of the pulsations; l is the scale of turbulence; c, c_1 and k are empirical constants; δ_{ij} is Kronecker symbol; i, j = 1, 2, 3.

Replacing the first equation of the system (1.1)-(1.6) by the sum of the first three equations, and the second by the sum of the second and third equations, we convert it with consideration of equations (1.7) and (1.8) and the designators for local criteria[1]

$$R_l = \frac{l^2}{\nu} \frac{d\langle v_\varphi \rangle}{dr}, \quad R_{\omega l} = \frac{l^2}{\nu} \frac{\langle v_\varphi \rangle}{r}, \quad R_E = \frac{l}{\nu} \sqrt{E}$$

---

[1] Criterion

$$R_e = \frac{l^2}{\nu} \frac{d\langle v_\varphi \rangle}{dr}$$

is analogous to the local Reynolds criterion

$$R = \frac{l^2}{\nu} \frac{d\langle v_x \rangle}{dy}$$

first introduced in the works of L. G. Loytsyanskiy [9].

to the nondimensional form

$$\frac{\langle v_r' v_\varphi' \rangle}{E} R_l - \frac{\langle v_r' v_\varphi' \rangle}{E} R_{\omega l} + CR_E + C_1 = 0 \tag{1.9}$$

$$2\frac{\langle v_r' v_\varphi' \rangle}{E} R_l - 2\frac{\langle v_r' v_\varphi' \rangle}{E} R_{wl} + \frac{(\langle v_r'^2 \rangle + \langle v_\varphi'^2 \rangle)}{E}(kR_E + C_1) - \frac{4}{3}(k-c) R_E = 0 \tag{1.10}$$

$$2\frac{\langle v_r' v_\varphi' \rangle}{E} R_l + 2\frac{\langle v_r' v_\varphi' \rangle}{E} R_{wl} + \frac{\langle v_\varphi'^2 \rangle}{E}(kR_E + c_1) - \frac{2}{3}(k-c) R_E = 0 \tag{1.11}$$

$$\frac{\langle v_x' v_r' \rangle}{E}(kR_E + c_1) - 2\frac{\langle v_x' v_\varphi' \rangle}{E} R_{wl} - 0 \tag{1.12}$$

$$\frac{\langle v_x' v_r' \rangle}{E}(R_l + R_{wl}) + \frac{\langle v_x' v_\varphi' \rangle}{E}(kR_E + c_1) = 0 \tag{1.13}$$

$$\frac{\langle v_r' v_\varphi' \rangle}{E}(kR_E + c_1) + \frac{\langle v_r'^2 \rangle}{E}(R_l + R_{wl}) - \frac{\langle v'^2 \rangle}{E} 2R_{wl} = 0 \tag{1.14}$$

Note that (1.9) represents an equation of equilibrium of the whole energy of turbulence in the flow. As expected in this equation, generation and dissipation of energy play a fundamental role in the equilibrium of turbulent energy in the approximation under examination. These values are approximately identical over a large portion of the channel cross section, so that turbulence is practically found in a state of energy balance.

System (1.9) through (1.14) consists of six equations and contains eight unknown ($\langle v_i' v_j' \rangle$, $\langle v_\phi \rangle$ and l). In this connection it is necessary to add the equation for average flow

$$\frac{\partial}{\partial r}(\langle v_r' v_\varphi' \rangle r^2) = \nu \frac{\partial}{\partial r}\left[r^3 \frac{\partial (\langle v_\varphi \rangle / r)}{\partial r}\right]$$

which we write, after single integration and utilization of the designations given earlier,

$$R_{wl} - R_l + \frac{\langle v_r' v_\varphi' \rangle}{E} R_E^2 = \left(\frac{v_{*i} l}{\nu}\right)^2 \left(\frac{r_i}{r}\right)^2 \tag{1.15}$$

and determine the scale of turbulence l.

Solving the system of equations (1.9)-(1.14), we readily obtain /123

$$\langle v_\varphi'^2 \rangle / E = \frac{^2/_3 (k-c) R_E}{(kR_E + C_1)} - \frac{2(cR_E + c_1)(R_l + R_{wl})}{(R_{wl} - R_l)(kR_E + c_1)} \tag{1.16}$$

$$\langle v_r'^2\rangle / E = \frac{{}^4/_3 (k-c) R_E R_{wl}}{(kR_E + c_1)(R_l + R_{wl})} + \frac{4(cR_E + c_1) R_{wl}}{(R_l - R_{wl})(kR_E + c_1)} - \frac{(CR_E + c_1)(kR_E + c_1)}{(R_{wl} - R_l)(R_{wl} + R_l)} \tag{1.17}$$

$$\frac{\langle v_x'^2\rangle}{E} = 2 - \frac{\langle v_r'^2\rangle}{E} - \frac{\langle v_\varphi'^2\rangle}{E} \tag{1.18}$$

$$\langle v_x' v_r'\rangle = \langle v_x' v_\varphi'\rangle = 0, \quad \frac{\langle v_r' v_\varphi'\rangle}{E} = \frac{CR_E + c_1}{R_{wl} - R_l} \tag{1.19}$$

$$\frac{{}^2/_3 (k-c) R_E (R_{wl} - R_l)^2}{8R_{wl}(R_l + R_{wl}) + (kR_E + c_1)^2} = cR_E + c_1 \tag{1.20}$$

The left and right-hand sides of (1.20) represent the generation and dissipation of pulsation energy in abstract form.

2. The Region of Friction Near the Channel Walls. Near the channel walls, the inequality $\langle v_\phi\rangle/r \ll d\langle v_\phi\rangle/dr$ is valid, and consequently $R_{wl} \ll R_l$. Using these inequalities, we convert the expressions (1.16)-(1.20) to the form

$$v_{\varphi i}^* = \left[\frac{{}^2/_3 (k/c - 1) R_E}{(k/c) R_E + c_1/c} + \frac{2(R_E + c_1/c)}{(k/c) R_E + c_1/c}\right]^{1/2} \frac{R_E}{\eta_i^+} \tag{2.1}$$

$$v_{ri}^* = \left[\frac{{}^2/_3 (k/c - 1) R_E}{(k/c) R_E + c_1/c}\right]^{1/2} \frac{R_E}{\eta_i^+} \tag{2.2}$$

$$v_{xi}^* = \left[2 - \frac{{}^4/_3 (k/c - 1) R_E}{(k/c) R_E + c_1/c} - \frac{2(R_E + c_1/c)}{(k/c) R_E + c_1/c}\right]^{1/2} \frac{R_E}{\eta_i^+} \tag{2.3}$$

$$\langle v_x' v_r\rangle = \langle v_x' v_\varphi'\rangle = 0, \quad v_i^{**} = -\frac{CR_E + c_1}{R_l} \frac{R_E^2}{(\eta_i^+)^2} \tag{2.4}$$

$$\frac{{}^2/_3 (k-c) R_E R_l^2}{(kR_E + c_1)^2} = cR_E + c_1 \tag{2.5}$$

We also express through R_E, R_e and η_i^+, the values

$$E^* = \frac{R_E}{\eta_i^+}, \quad \frac{d\varphi}{d\eta_i} = (-1)^{i+1} \frac{R_l}{(\eta_i^+)^2} \quad (i = 1, 2) \tag{2.6}$$

Here $\phi = \langle v_\phi\rangle / v_{*i}$; $v_{*i} = \sqrt{\tau_i/\rho}$ is the dynamic velocity, τ_1, τ_2 represent the force of friction on the rotating and stationary walls of the channel:

$$v_{ij}^* = \frac{\sqrt{\langle v_j'^2\rangle}}{v_{*i}} \quad (j = \varphi, r, x); \quad v_i^{**} = \frac{\langle v_r' v_\varphi'\rangle}{v_{*i}^2}$$

$$E^* = \frac{\sqrt{E}}{v_{*i}}, \quad \eta_i^+ = \frac{l v_{*i}}{\nu}, \quad \eta_i = \frac{y v_{*i}}{\nu}$$

Assuming that

$$\langle v_r' v_\varphi' \rangle = -\varepsilon \frac{d\langle v_\varphi \rangle}{dy} \tag{2.7}$$

(ε is the coefficient of turbulent kinematic viscosity); considering (2.4), we obtain

$$\frac{\varepsilon}{\nu} = \frac{(cR_E + c_1) R_E^2}{R_l^2} \tag{2.8}$$

In the region of turbulent flow developing near the walls, we write /124
the correlation $\langle v_r' v_\phi' \rangle$, after discarding the component in (2.4) related to the effect of physical viscosity ($c_1 = 0$), and using (2.5), as

$$-\rho \langle v_r' v_\varphi' \rangle = \frac{[2/3 (k/c - 1)]^{3/2}}{k^3/c} \rho l^2 \left| \frac{d\langle v_\varphi \rangle}{dr} \right| \frac{d\langle v_\varphi \rangle}{dr} \tag{2.9}$$

In connection with the fact that the scale of turbulence l is defined in (1.7) and (1.8) with accuracy up to a constant multiplier, on whose selection the values of constants k, c and c_1 depend, we take [8]

$$\frac{2/3 (k/c - 1)^{3/2}}{k^3/c} = 1 \tag{2.10}$$

In this case (2.9) agrees with the known Prandtl formula after which one may also put $l = \kappa y$, where $\kappa = 0.4$. Function (2.10) enables us to replace coefficients k and c with their ratio k/c

$$k^2 = \frac{[2/3 (k/c - 1)]^{3/2}}{k/c}, \qquad c^2 = \frac{[2/3 (k/c - 1)]^{3/2}}{(k/c)^3}$$

and at the same time reduce the quantity of empirical constants.

For calculation of the basic characteristics of turbulent circular flow from (2.1) through (2.8) it is essential to establish the function of quantities R_E, R_e and η_i^+ to coordinate η_i. Here the connection between R_E and R_e, obtained from (2.5), calculating by (2.10), has the form

$$R_l = -c^{1/3} \left(R_E - \frac{c_1}{k} \right) \left[\frac{R_E + c_1/c}{R_E} \right]^{1/2} \tag{2.11}$$

In Figure 1 (Curves I and II) the results of the values of constants k/c = 7 and $c_1 = 2.45$ as calculated by (2.11) are plotted as selected, as well as from [8], which relate to flow in a straight tube.

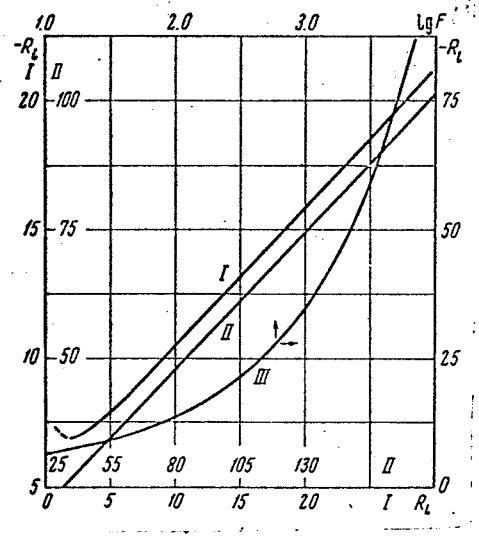

Figure 1. Criterion R_e vs. R_E (Curves I and II) and F (Curve III).

Substituting the value $\langle v_r' v_\phi' \rangle$ from (2.4) in (1.15) and calculating (2.11), we convert it to the form

$$R_l + \frac{R_E^2 (CR_E + c_1)}{R_l} = -\varkappa^2 \eta_i^2 \qquad (2.12)$$

Knowing the connection between criteria R_e and R_E we present the left-hand side of (2.12) in the form of a function of R_e:

$$R_l + \frac{R_E^2 (cR_E + c_1)}{R_l} = F(R_l) \qquad (2.13)$$

as shown in Figure 1 (Curve III). Then, finding from correlation

$$-\varkappa^2 \eta_i^2 = F(R_e) \qquad (2.14)$$

the distribution of criterion R_l in the cross section of the channel, it is easy to calculate from (2.1)-(2.8) the rest of the characteristics of flow as well, including the velocity profile near the wall.

3. Turbulent Core of Flow. In the turbulent core of flow, in which motion is near the potential circulation [5], we adopt the law

$$\langle v_\varphi \rangle r = c_0 v_1 r_1 = \text{const} \quad (c_0 \approx 0.55) \qquad (3.1)$$

(v_1 is the velocity of rotation of the cylinder), average eddying of flow $1/r\ (d\langle v_\phi \rangle r/dr)$ is extremely small, and consequently, one can propose

$$R_l + R_{\omega l} = \frac{l^2}{\nu} \frac{1}{r} \frac{d}{dr} (\langle v_\varphi \rangle r) \approx 0 \qquad (3.2)$$

On the other hand, in the turbulent core of flow one can discard terms relating to the motion of the force of viscosity ($c_1 \approx 0$). In this case, (1.16)-(1.20) are written /125

$$v_{\varphi i}^* = k^{1/2} c^{1/4} \frac{R_E}{\eta_i^+} \qquad (3.3)$$

$$v_{ri}^{*}=\left[kc^{1/3}-\frac{4c^{2/3}R_{wl}}{kR_E}\right]^{1/2}\frac{R_E}{\eta_i^{+}} \tag{3.4}$$

$$v_{xi}^{*}=\left[2-2kc^{1/3}+\frac{4c^{2/3}R_{wl}}{kR_E}\right]^{1/2}\frac{R_E}{\eta_i^{+}} \tag{3.5}$$

$$\langle v_x'v_r'\rangle=\langle v_x'v_\varphi'\rangle=0,\qquad v_i^{**}=c^{2/3}\frac{R_E^2}{(\eta_i^{+})^2} \tag{3.6}$$

$$R_{wl}-R_l=c^{1/3}R_E \tag{3.7}$$

$$E^{*}=\frac{R_E}{\eta_i^{+}} \tag{3.8}$$

The equation for average motion (1.5) for turbulent core of flow, after substituting the values $\langle v_r'v_\phi'\rangle$ and R_E from (3.6) and (3.7), will have the form

$$c^{1/3}R_E=\frac{\eta_i^{+}}{r/r_i} \tag{3.9}$$

From (3.6) and (3.7) we easily obtain the expression for the force of friction:

$$\tau=-\langle\rho v_r'v_\varphi'\rangle=\rho l^2\left|\frac{d\langle v_\varphi\rangle}{dr}-\frac{\langle v_\varphi\rangle}{r}\right|\left(\frac{d\langle v_\varphi\rangle}{dr}-\frac{\langle v_\varphi\rangle}{r}\right)=\rho\varepsilon\left(\frac{d\langle v_\varphi\rangle}{dr}-\frac{\langle v_\varphi\rangle}{r}\right) \tag{3.10}$$

which for $\langle v_\phi\rangle/r \ll d\langle v_\phi\rangle/dr$ (in the sloping region) converts to the previously obtained formula (2.9).

In accordance with [10], we propose that the scale of turbulence l in the developing turbulent circular flow is proportional to radius

$$l=\alpha r,\qquad \alpha=\frac{v_{*1}}{2c_0v_1} \tag{3.11}$$

Here the constant α is determined from (3.1), (3.9), and (3.10).

Using (3.7)-(3.11) we write (3.3)-(3.8) in the final form:

$$v_{\varphi i}^{*}=k^{1/2}c^{-1/6}\frac{1}{r/r_i} \tag{3.12}$$

$$v_{ri}^{*}=\left[kc^{1/3}-\frac{2c}{k}\right]^{1/2}\frac{c^{-1/3}}{r/r_i} \tag{3.13}$$

$$v_{xi}^{*}=\left[2-2kc^{1/3}+\frac{2c}{k}\right]^{1/2}\frac{c^{-1/3}}{r/r_i} \tag{3.14}$$

$$v_i^{**}=\frac{1}{(r/r_i)^2} \tag{3.15}$$

$$E^{*}=\frac{c^{-1/3}}{r/r_i} \tag{3.16}$$

We will define the coefficient of turbulent viscosity from (3.10), substituting the value of l and $\langle v_\phi \rangle$ in it from (3.11) and (3.1):

$$\frac{\varepsilon}{\nu} = \frac{1}{2c_0} \frac{R^+}{v_1/v_{*1}} \qquad \left(R^+ = \frac{v_{*1} r_1}{\nu} = \frac{v_{*2} r_2}{\nu}\right) \tag{3.17}$$

Integrating (3.9) with calculation of function (3.11), we obtain the distribution of average velocity in the turbulent core of the stream /126

$$\frac{\varphi}{R^+ + (-1)^{i+1}\eta_i} = \frac{\varphi^*}{R^+ + (-1)^{i+1}\eta_i^*} - c_0 R_0 \left\{ \frac{1}{[R^+ - (-1)^{i+1}\eta_i^*]^2} - \frac{1}{[R^+ - (-1)^{i+1}\eta_i]^2} \right\} \tag{3.18}$$

Here $R_0 = v_1 r_1/\nu$; $\eta_i^* = y^* v_{*i}/\nu$ is the value of the common coodinate at the conditional boundary of transition layer $r^* = r_i + (-1)^{i+1} y^*$, $(i = 1, 2)$: ϕ^* is the corresponding velocity of this boundary.

4. Experimental Agreement. Comparison of the calculated functions with the results of experimental study of the hydrodynamic turbulent circular flow in channels with a central rotating cylinder [5, 11] is shown in Figures 2 through 6.

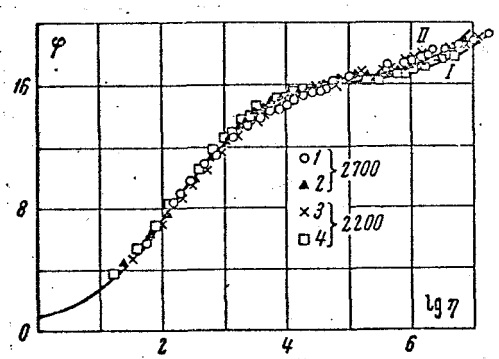

Figure 2. Averaged Curve of Abstract Velocity $\phi(\eta)$: solid lines are calculated values; I and II are calculation by (3.18) for i = 1 and 2; 1, 3 are measurements near a rotating wall; 2, 4 are the same near a stationary one.

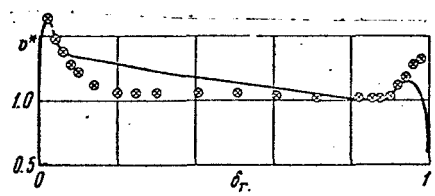

Figure 3. Distribution of Pulsation Intensity of the Tangential Velocity Component in the Channel Cross Section.

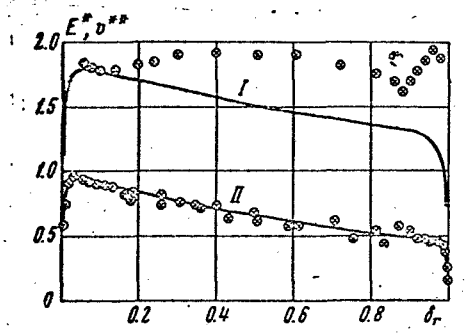

Figure 4. Distribution of Total Energy of Turbulence (Curve I) and Correlation in the Channel Cross Section (Curve II).

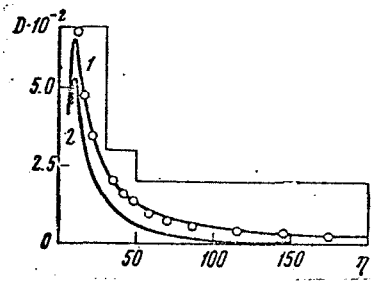

Figure 6. Distribution of the Components of Equilibrium of Pulsation Energy in the Flow Region at the Rotating Wall. Solid lines--theoretical curves; 1, generation--dissipation; 2, contribution to dissipation due to effect of viscosity on large scale pulsation motion.

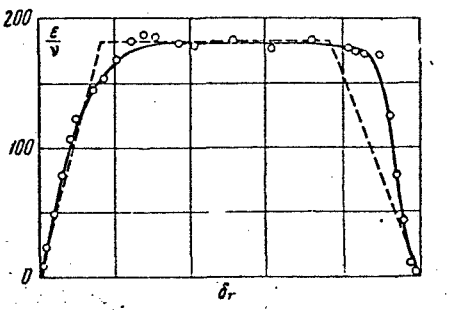

Figure 5. Distribution of the Coefficient of Turbulent Viscosity in the Channel Cross Section: solid line--experimental; dotted line--calculated.

In Figure 2 the universal curve of velocity ϕ is illustrated, constructed from (2.6) and (3.18), and it is in good agreement with the experiment. Here, as in [8], division of the flow into a laminar sublayer, transition region and turbulent core does not require special assumptions, but is the outcome of the resultant system of equations.

Distribution in the cross section of the channel of the pulsation intensity component of velocity vector v_{ij}*, total turbulent energy E*, correlation v_i** and coefficient of turbulent viscosity ε/ν, determined by (2.1)-(2.8) and (3.12)-(3.17) are also in complete satisfactory agreement with the experimental data (Figures 3-5).

Figure 6 presents the distribution of balance components of pulsation energy near a rotating wall. An analogous picture occurs at a stationary wall. In addition to the good agreement of the calculated curve with the experimental one, note the close correspondence of the picture obtained of the distribution with the analogous picture in rectilinear flows.

REFERENCES

1. Koshmarov, Yu. A., "Hydrodynamics and Thermal Exchange of Flow of a Non-compressed Liquid in the Space Between Rotating Coaxial Cylinders," *Inzh.-Fiz.*, Vol. 5, No. 5, pp. 5-14, 1962.
2. Gazley, C., "Heat--Transfer Characteristics of the Rotational and Axial Flow Between Concentric Cylinders," *Trans. ASME*, Vol. 80, No. 1, pp. 79-90, 1958.
3. Dorfman, L. A., *Gidrodinamicheskoye soprotivleniye i teplootdacha vraschayuschikhcya tel*, [Hydrodynamic Resistance and Thermal Exchange of Rotating Bodies], Fizmatgiz Press, Moscow, 1960. /127
4. Leont'ev, A. I., and A. G. Kirdyashkin, "Friction and Heat Exchange in the Space Between Two Rotating Coaxial Cylinders," *Inzh.-Fiz. zh.*, Vol. 13, No. 6, pp. 821-830, 1967.
5. Ustimenko, B. P., and V. N. Zmeykov, "Hydrodynamics of Flow in a Circular Channel with Internal Rotating Cylinder," *Teplofizika vysokikh temperatur*, Vol. 2, No. 2, pp. 250-259, 1964.
6. Rotta, I., "Statistical Theory of Inhomogeneous Turbulence," *Z. fur Phys.*, Vol. 129, No. 6, pp. 547-572, 1951.
7. Levin, V. B., "Stabilizing Effect of Rotating a Flow on Turbulence," *Teplofizika vysokikh temperatur*, Vol. 2, No. 6, pp. 892-900, 1964.
8. Levin, V. B., "Calculation of Fundamental Characteristics of Turbulent Flows with Transverse Motion," *Teplofizika vysokikh temperatur*, Vol. 2, No. 4, pp. 588-598, 1964.
9. Loytsyanskiy, L. G., "Hypothesis of Locality in Turbulent Motion of Liquids Having Viscosity," *PMM*, 5th edition, Vol. 22, pp. 600-611, 1958.
10. Kinney, R. B., "Universal Velocity Similarity in Fully Turbulent Rotating Flow," *Trans. ASME, Ser. E., J. Appl. Mech.*, Vol. 34, No. 2, 1967.
11. Ustimenko, B. P., and V. N. Zmeykov, "Turbulent Structure of Circular Flow in Circumferential Channels Between Rotating Coaxial Cylinders," *Teplofizika vysokikh temperatur*, Vol. 5, No. 4, pp. 640-646, 1967.

Translated for the National Aeronautics and Space Administration under contract No. NASw-2037 by Techtran Corporation, P. O. Box 729, Glen Burnie, Maryland, 21061.